CS 70 Discrete Mathematics and Probability Theory
Fall 2025 Sabin, Hug

DIS 11A

1 Calculate These... or Else

Note 14

(a) A straight is defined as a 5 card hand such that the card values can be arranged in consecutive ascending order (i.e. $\{8,9,10,J,Q\}$ is a straight). Values do not loop around, so $\{Q,K,A,2,3\}$ is not a straight. However, an ace counts as both a low card and a high card, so both $\{A,2,3,4,5\}$ and $\{10,J,Q,K,A\}$ are considered straights.

When drawing a 5 card hand, what is the probability of drawing a straight from a 52-card deck?

(b) What is the probability of drawing a straight or a flush? (A flush is five cards of the same suit.)

(c) When drawing a 5 card hand, what is the probability of drawing at least one card from each suit?

(d) Two distinct squares are chosen at random on an 8×8 chessboard. What is the probability that they share a side?

(e) 8 rooks are placed randomly on an 8×8 chessboard. What is the probability none of them are attacking each other? (Two rooks attack each other if they are in the same row, or in the same column.)

2 Box of Marbles

Note 14

You are given two boxes: one of them containing 900 red marbles and 100 blue marbles, the other one contains 500 red marbles and 500 blue marbles.

(a) If we pick one of the boxes randomly, and pick a marble what is the probability that it is blue?

(b) If we see that the marble is blue, what is the probability that it is chosen from box 1?

(c) Suppose we pick one marble from box 1 and without looking at its color we put it aside. Then we pick another marble from box 1. What is the probability that the second marble is blue?

3 Professional Crastination

Note 18
Note 19

You are a college student with a busy life, so your CS70 homework may be pushed aside. Each day, you have a 0.24 probability of working on your CS70 homework (once), independent of other days.

(a) On average, how many days does it take **before** you start working on homework *for the first time*?

(b) In the last 7 days, what is the probability that you do *not* work on your homework every day?

(c) You notice that your sleep schedule is suboptimal, and each *hour*, you have a 0.01 probability of working on your CS70 homework (once), independent of other hours.

In the last 7 days, how many times do you work on your homework, on average? (Recall that there are 24 hours in a day.)

(d) Every 7 days, you work on a new homework assignment. What is the probability that you do not work on *all four* of the last four homework assignments (at least once)?

4 Balls in Bins

Note 19

You are throwing k balls into n bins. Let X_i be the number of balls thrown into bin i.

(a) What is $\mathbb{E}[X_i]$?

(b) What is the expected number of empty bins?

(c) Define a collision to occur when a ball lands in a nonempty bin. What is the expected number of collisions? *Hint: If there are n balls in a bin, $n-1$ collisions have occurred in that bin.*

5 Diversify Your Hand

Note 19
Note 20

You are dealt 5 cards from a standard 52 card deck. Let X be the number of distinct values in your hand. For instance, the hand (A, A, A, 2, 3) has 3 distinct values.

(a) Calculate $\mathbb{E}[X]$. (Hint: Consider indicator variables X_i representing whether i appears in the hand.)

(b) Calculate $\text{Var}(X)$. The answer expression will be quite involved; you do not need to simplify anything.