

Due: Saturday, 9/6, 4:00 PM
Grace period until Saturday, 9/6, 6:00 PM
Remember to show your work for all problems!

Sundry

Before you start writing your final homework submission, state briefly how you worked on it. Who else did you work with? List names and email addresses. (In case of homework party, you can just describe the group.) If you used an LLM, place transcripts of your chats here.

1 Solving a System of Equations Review

Alice wants to buy apples, beets, and carrots. An apple, a beet, and a carrot cost 16 dollars, two apples and three beets cost 23 dollars, and one apple, two beets, and three carrots cost 35 dollars. What are the prices for an apple, for a beet, and for a carrot, respectively? Set up a system of equations and show your work.

2 Calculus Review

In the probability section of this course, you will be expected to compute derivatives, integrals, and double integrals. This question contains a couple examples of the kinds of calculus you will encounter.

- (a) Compute the following integral:

$$\int_0^{\infty} \sin(t) e^{-t} dt.$$

- (b) Compute the double integral

$$\iint_R 2x + y \, dA,$$

where R is the region bounded by the lines $x = 1$, $y = 0$, and $y = x$.

3 Logical Equivalence?

Note 1

Decide whether each of the following logical equivalences is correct and justify your answer.

- (a) $\forall x (P(x) \wedge Q(x)) \stackrel{?}{=} \forall x P(x) \wedge \forall x Q(x)$

(b) $\forall x (P(x) \vee Q(x)) \stackrel{?}{=} \forall x P(x) \vee \forall x Q(x)$

(c) $\exists x (P(x) \vee Q(x)) \stackrel{?}{=} \exists x P(x) \vee \exists x Q(x)$

(d) $\exists x (P(x) \wedge Q(x)) \stackrel{?}{=} \exists x P(x) \wedge \exists x Q(x)$

4 If, Then

Note 2

Consider the conjecture: The sum of an even integer and an odd integer is odd.

- (a) Write the conjecture as an if/then statement.
- (b) An if/then proof always starts with an assumption. What can you assume here to begin the proof?
- (c) Based on your assumption, what do you Want To Show that would complete the proof?
- (d) Complete the proof of the conjecture.

5 Prove

Note 2

Prove each of the following statements

- (a) $\forall a, b, c \in \mathbb{Z}$, if $a|b$ and $b|c$, then $a|c$.
- (b) $\forall n \in \mathbb{N}$, n is odd if and only if $3n + 5$ is even
- (c) $\forall n \in \mathbb{Z}$, $n^2 + n + 6$ is even.

6 Prove or Disprove

Note 2

For each of the following, either prove the statement, or disprove by finding a counterexample.

- (a) $(\forall n \in \mathbb{N})$ if n is odd then $n^2 + 4n$ is odd.
- (b) $(\forall a, b \in \mathbb{R})$ if $a + b \leq 15$ then $a \leq 11$ or $b \leq 4$.
- (c) $(\forall r \in \mathbb{R})$ if r^2 is irrational, then r is irrational.
- (d) $(\forall n \in \mathbb{Z}^+) 5n^3 > n!$. (Note: \mathbb{Z}^+ is the set of positive integers)
- (e) The product of a non-zero rational number and an irrational number is irrational.

7 The Tell of the Digits

Note 2

Prove the following.

For all four-digit natural numbers, if the alternating sum (alternating subtracting and adding) of its digits is divisible by 11, then the number itself is divisible by 11.

For example, for the number 5291, the alternating sum of its digits is $5 - 2 + 9 - 1 = 11$ is divisible by 11 and, indeed, 5291 is itself divisible by 11. Yet 9718 has an alternating sum of $9 - 7 + 1 - 8 = -5$ is not divisible by 11 and so there is no guarantee on whether 9718 is divisible by 11 (and, in fact, it does not evenly divide it).

8 (Optional) Study Groups

This semester, we're facilitating the formation of study groups for students to collaborate on the homework and study together otherwise.

We especially want to encourage study groups to come to office hours together and receive support from our course staff. **Maybe: Study groups will given an opportunity to book a regular time at Wednesday and Thursday office hours to work on the HW together with staff support.**

If you're interested in being assigned a study group by us, please fill out this form: <https://docs.google.com/forms/d/e/1FAIpQLSdEblrc1r397szWCugMdwe7QRFYBgC2ru2aACmFH1e110Ey6Q/viewform?usp=header> by this homework's due date.